

# Multiphase Cooling Flows

Peter A. Thomas

*Astronomy Centre, Dept. of Physics and Astronomy, University of  
Sussex, Falmer, Brighton, BN1 9QH, UK*

**Abstract.** I discuss the multiphase nature of the intracluster medium whose neglect can lead to overestimates of the baryon fraction of clusters by up to a factor of two. The multiphase form of the cooling flow equations are derived and reduced to a simple form for a wide class of self-similar density distributions. It is shown that steady-state cooling flows are *not* consistent with all possible emissivity profiles which can therefore be used as a test of the theory. In combination, they provide strong constraints on the mass distribution within the cooling radius.

## 1. Introduction

The multiphase nature of the intracluster medium (icm) in cooling flows was demonstrated a decade ago when deprojections of X-ray surface-brightness profiles showed that mass cools and is deposited from the flow in a distributed manner,  $\dot{M} \propto r$  (e.g. Thomas, Fabian & Nulsen 1987). However, the complexity of the theory and lack of data with high spatial resolution means that the single-phase approximation is still widely adopted. In this paper I attempt to present the theory in a palatable form and give examples of its application.

## 2. The multiphase cooling flow equations

The theory of multiphase cooling flows was set out by Nulsen (1986). I rederive the equations in a slightly different form here in the hope that they may prove more accessible.

### 2.1. Derivation of the equations

We assume an emulsion of density phases which comove with the flow. The distribution is described by the volume fraction,  $f(\rho, \mathbf{r}, t)$ , such that  $f d\rho$  is the fractional volume occupied by phases with densities in the range  $\rho$  to  $\rho + d\rho$ . Then  $\int f d\rho = 1$ , and the mean density is  $\bar{\rho} = \int f \rho d\rho$ .

Mass conservation gives

$$\frac{\partial}{\partial t}(\rho f) + \nabla \cdot (\mathbf{u} \rho f) + \frac{\partial}{\partial \rho}(\dot{\rho} \rho f) = 0, \quad (1)$$

where  $\mathbf{u}$  is the rate of change of position and  $\dot{\rho}$  is the rate of change of density following the flow. The final term in Equation 1 is the equivalent in density space of the divergence in velocity space.

Integrating over all densities we obtain

$$\frac{\dot{\bar{\rho}}}{\bar{\rho}} + \nabla \cdot \mathbf{u} + \beta = 0, \quad (2)$$

where

$$\beta \equiv \frac{1}{\bar{\rho}} \lim_{\rho \rightarrow \infty} (\dot{\rho} \rho f). \quad (3)$$

This is equivalent to the usual single-phase equation (e.g. Thomas 1988a) except that the mass deposition rate is specified in terms of  $f$  rather than being a free parameter.

To find how the volume fraction changes with time we use the energy equation,

$$\frac{\dot{\rho}}{\rho} = \frac{1}{\gamma} \frac{\dot{P}}{P} + \frac{\gamma - 1}{\gamma} \frac{n^2 \Lambda}{P}, \quad (4)$$

where  $P$  is the pressure,  $n^2 \Lambda$  is the radiated power per unit volume and  $\gamma = 5/3$  for an ionised plasma.

Over a wide temperature range appropriate to clusters the cooling function can be approximated by a power-law,  $\Lambda \propto T^\alpha$ , where  $\alpha \approx 0.5$ . Then Equation 4 can be simplified by moving to a new density variable. Writing

$$\rho = \rho_0(\mathbf{r}, t) w^{-1/(2-\alpha)}, \quad (5)$$

we obtain

$$\dot{w} = (2 - \alpha) \left( \frac{\dot{\rho}_0}{\rho_0} - \frac{1}{\gamma} \frac{\dot{P}}{P} \right) w - (2 - \alpha) \frac{\gamma - 1}{\gamma} \frac{n_0^2 \Lambda(T_0)}{P}. \quad (6)$$

If the adiabatic compression term is removed by setting  $P \propto \rho_0^\gamma$ , then the energy equation takes a particularly simple form,  $\dot{w} = \text{constant}$ . However, a more useful choice is to take  $\rho_0 \propto \bar{\rho}$ .

From the final term of Equation 1, we see that at high density when cooling is dominant, then  $\dot{\rho} \rho f \sim \text{constant}$ . This motivates the substitution

$$f = \frac{(2 - \alpha)}{\rho_0} w^{(4-\alpha)/(2-\alpha)} g(w, \mathbf{r}, t). \quad (7)$$

Then, using Equations 1, 4 and 6 we obtain the following equation for the covariant derivate of  $g$  (i.e. following the fluid flow):

$$\frac{\dot{g}}{g} + (3 - \alpha) \frac{\dot{\rho}_0}{\rho_0} - \frac{2 - \alpha}{\gamma} \frac{\dot{P}}{P} + \nabla \cdot \mathbf{u} = 0. \quad (8)$$

## 2.2. The form of the density distribution

In general  $g$  is a complicated function of position and time. However, we can look for solutions in which  $g$  has a constant functional form,  $g = g(w)$ . Only the first term in Equation 8 depends upon  $w$ . Hence we require that  $\dot{g} \equiv \dot{w} dg/dw \propto g$ . There are two kinds of solution:

1.  $g_\infty \propto \exp(-w)$ . This is the most extended distribution which is convectively stable (it gives  $P \propto \rho_0^\gamma$ ). It includes phases of arbitrarily low density.
2.  $g_k \propto (1-w)^{k-1}$ ,  $0 < w < 1$ ;  $k \geq 1$ . These solutions possess a minimum density,  $\rho \geq \rho_0$ .  $k = 1$  is the least extended, consisting solely of the power-law cooling tail. As  $k \mapsto \infty$  the solutions resemble  $g_\infty$ .

For other forms of  $g$  we must resort to numerical integration to follow their evolution. Thomas (1988b) looked at the steady-state evolution of a range of distributions with a sharp cut-off at low densities and reached the following conclusions:

- All distributions develop a high-density tail,  $f \sim \rho^{-(4-\alpha)}$ , as they cool.
- Sufficiently narrow distributions resemble the pure power-law  $g_1$  by the time they begin to be deposited.

$g_1$  and  $g_\infty$  bound all reasonable solutions of the cooling flow equations, be they self-similar in form or not. I would also argue that plausible formation histories for the icm make low values of  $k$  more likely than high values.

### 3. The Baryon Catastrophe

Modelling of the icm suggests that the baryon fraction in clusters is of order  $0.06 h^{-1.5}$  or more (e.g. White et al. 1993, White & Fabian 1995), where  $h = H_0/100 \text{ km s}^{-1} \text{ Mpc}^{-1}$  is the dimensionless Hubble parameter. However, primordial nucleosynthesis limits baryons to a small fraction of the critical density,  $\Omega_b < 0.015 h^{-2}$  (Walker et al. 1991). If clusters contain a representative mixture of material, then it would seem that the Universe must be open with density parameter  $\Omega \lesssim 0.25 h^{-0.5}$ . This has been termed the Baryon Catastrophe.

One flaw in the above argument is that the modelling is based on single-phase models of the icm. We know that there must be a wide range of densities at the edge of the cooling flow and it is plausible that the same is true throughout the cluster. Moving to a multiphase model can mitigate (but not eliminate) the Baryon Catastrophe.

First let me outline the determination of the gas and total masses of a cluster in the single-phase case. The equation of hydrostatic support relates the mass of a spherical cluster to the properties of its icm:

$$\frac{GM(< r)}{r} = -\frac{1}{\rho} \frac{dP}{dr}, \quad (9)$$

where  $G$  is the gravitational constant and  $M$  is the total mass within radius  $r$ . To determine  $M$  we need to know both the density and pressure as a function of radius, whereas we usually have only one constraint: the surface brightness profile can be deprojected to give the emissivity,  $\xi(r) = n^2 \Lambda$ . To proceed we assume a polytropic relation,  $P \propto \rho^\Gamma$  where  $1 \leq \Gamma \leq \frac{5}{3}$  and normalise the average, emission-weighted temperature,  $T_X$ , to some observed value.

When moving to a multiphase model the same procedure holds, except that we have to average over all density phases. Hence  $\rho \mapsto \bar{\rho} = \rho_0 \int g \, dw$ ,

$$\xi \mapsto n^2 \Lambda = n_0^2 \Lambda(T_0) \int w^{-(1-\alpha)/(2-\alpha)} g \, dw, \quad (10)$$

and so on. The details of these calculations can be found in Gunn & Thomas (1996). Suffice it to say that the form of the solutions is unchanged but the gas mass is lowered and the total mass increased over the single-phase case. For  $\alpha = 0.5$  the baryon fractions are lower by factors of 0.74 and 0.60 for the distributions  $g_1$  and  $g_\infty$ , respectively (these factors decrease slightly at lower temperatures for which  $\alpha$  is lower).

Finally, let me note that most models ignore complicating factors such as magnetic fields and turbulence. Both of these can act to support the gas, thus raising the pressure in Equation 9 and lowering the gas fraction even further. It is too early to conclude that only low values of  $\Omega$  are compatible with the data.

#### 4. Reconstruction of cluster mass profiles

The self-similar density distributions derived in Section 2 lead to particularly simple forms of the steady-state cooling flow equations. I derive these below and then show how they can be combined with the emissivity profile to provide strong constraints of the mass distribution within the cooling radius.

##### 4.1. Theory

Substituting the functional form of  $g_k$  into Equations 2, 3 and 8 we see that the self-similar forms of the cooling flow equations are:

$$\frac{\dot{\rho}_0}{\rho_0} - \frac{1}{\gamma} \frac{\dot{P}}{P} - \frac{\beta}{(2-\alpha)k} = 0 \quad (11)$$

and

$$\frac{\dot{\rho}_0}{\rho_0} + \nabla \cdot \mathbf{u} + \beta = 0, \quad (12)$$

where

$$\beta = (2-\alpha)k \frac{\gamma-1}{\gamma} \frac{n_0^2 \Lambda(T_0)}{P}. \quad (13)$$

To these may be added the equation of hydrostatic support,

$$\nabla \Phi + \frac{\nabla P}{\bar{\rho}} = 0, \quad (14)$$

where  $\Phi$  is the gravitational potential. I assume here that the inflow is subsonic—this turns out to be a good approximation in all multiphase cooling flow models.

In a steady-state and spherical symmetry the above equations reduce to

$$\frac{d \ln P}{d \ln r} = -2\Sigma, \quad (15)$$

$$\frac{d \ln \bar{\rho}}{d \ln r} = -\frac{2}{\gamma} \Sigma - \frac{\tau}{2 - \alpha}, \quad (16)$$

and

$$\frac{d \ln u}{d \ln r} = -2 + \frac{2}{\gamma} \Sigma + \left( \frac{1}{2 - \alpha} + k \right) \tau, \quad (17)$$

where

$$\Sigma = \frac{GM}{2r} \cdot \frac{\mu m_H}{k_B T} \quad (18)$$

is the ratio of the virial to the thermal temperatures, and

$$\tau = (2 - \alpha) \frac{\gamma - 1}{\gamma} \frac{n_0^2 \Lambda(T_0)}{P} \frac{r}{u} = \frac{1}{k} \frac{d \ln \dot{M}}{d \ln r} \quad (19)$$

is the ratio of the inflow time to the constant-pressure cooling time.

Although there appear to be three equations here, the dimensionless ratios  $\Sigma$  and  $\tau$  are the only important variables. The third equation merely acts as a scaling (for fixed  $\tau$ ,  $\bar{\rho}^{2-\alpha} \propto p^{1-\alpha} u$ ). Hence the physics can be captured in just two equations:

$$\frac{d \ln \Sigma}{d \ln r} = \chi - 1 + 2 \frac{\gamma - 1}{\gamma} \Sigma - \frac{\tau}{2 - \alpha} \quad (20)$$

and

$$\frac{d \ln \tau}{d \ln r} = 3 - \frac{2}{\gamma} [(3 - \alpha) - \gamma(1 - \alpha)] \Sigma - \left( \frac{3 - \alpha}{2 - \alpha} + k \right) \tau \quad (21)$$

where  $\chi \equiv d \ln \dot{M} / d \ln r$ . The  $g_\infty$  equations can be recovered by letting  $k \mapsto \infty$  and using  $k\tau$  in place of  $\tau$  as the second dimensionless variable.

The usual way of proceeding is to pick a functional form for the mass profile,  $\chi$ , and then to solve for  $\Sigma$  and  $\tau$ . From this one can generate an emissivity profile,  $\xi(r)$ , for comparison with the data. Alternatively, we can specify  $\xi(r)$  and *determine* the mass profile. Suppose that  $\beta_{\text{fit}} \equiv -(1/6) d \ln \xi / d \ln r$  is known. Then

$$\frac{d \ln \tau}{d \ln r} = 3 - 6 \frac{(3 - \alpha) - \gamma(1 - \alpha)}{2 - \alpha + \alpha\gamma} \beta_{\text{fit}} - \left( \frac{2\gamma}{(2 - \alpha)(2 - \alpha + \alpha\gamma)} + k \right) \tau. \quad (22)$$

Furthermore this is an eigenvalue problem: requiring that the solution extend to  $r = 0$  fixes the outer boundary condition. Hence we can solve for  $\tau$ ,  $\Sigma$  and  $\chi$ .

We can get a good idea of the behaviour of the solutions by looking at the case of constant  $\beta_{\text{fit}}$ . Imposing the physical constraints  $\Sigma \geq 0$  (i.e. a non-negative temperature) and  $\chi \geq 0$  (i.e. mass constant or increasing with radius) restricts  $\beta_{\text{fit}}$  to lie in the range

$$\frac{3}{2(5 + 3k)} \leq \beta_{\text{fit}} \leq \frac{80 + 21k}{120 + 36k} \quad (23)$$

(in this expression and henceforth I set  $\gamma = 5/3$  and  $\alpha = 0.5$  rather than including them explicitly). Thus steep emissivity profiles,  $\beta_{\text{fit}} \gtrsim 0.65$ , are incompatible with all steady-state cooling flow models (larger values can occur, however, outside the cooling radius). In addition, The inner value of  $\beta_{\text{fit}}$  can be used to

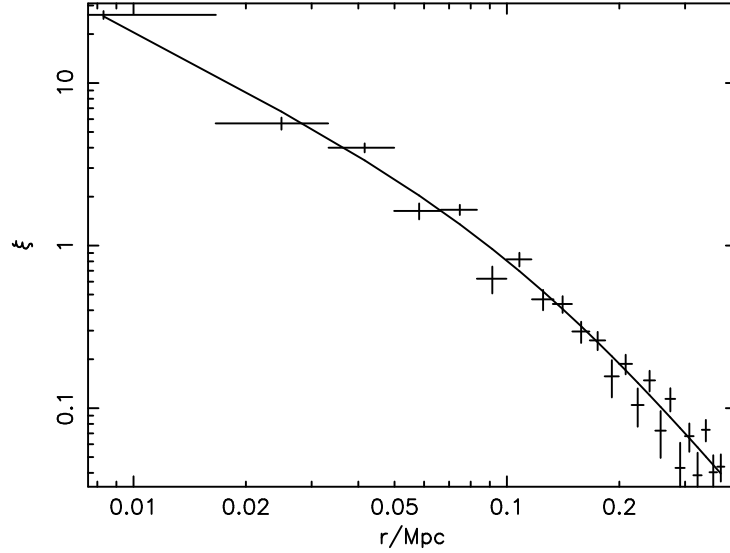


Figure 1. The emissivity profile for A85 in 12 arcsec bins. The solid line shows a broken power-law fit as described in the text.

constrain  $k$  or, if we assume that the virial temperature drops (i.e.  $\Sigma \mapsto 0$ ) within the cluster core, to measure it.

Note that the solution for  $\Sigma$  and  $\tau$  does not depend upon the normalisation of the gas and total gravitational masses. If desired, these can be determined by fixing the overall temperature and luminosity. The analysis of Section 3. shows that the gas density will be slightly lower and the total mass density slightly higher than in the equivalent single-phase analysis.

#### 4.2. Application to A85

I will now give an example of the application of the theory to the cooling flow in the cluster A85. The emissivity profile obtained with the ROSAT HRI (after correction for absorption and the instrumental energy response) were kindly supplied to me by Clovis Peres. It is shown in 12 arcsec bins in Figure 1 together with a simple broken power-law fit,

$$\xi \propto \left[ \left( \frac{r}{0.12 \text{ Mpc}} \right)^{1.18} + \left( \frac{r}{0.12 \text{ Mpc}} \right)^{2.83} \right]^{-1}. \quad (24)$$

The asymptotic slope of  $\xi$  as  $k \mapsto 0$  is very close to the minimum permitted for  $k = 1$  which suggests that the solution will require an inner core in the mass distribution. This is illustrated in Figure 2. Note that  $\Sigma$  drops very close to zero at  $r = 10 \text{ kpc}$  (it tends to a small constant value within this radius). The gravitational density profile (i.e. that of the total mass, not just the gas) is

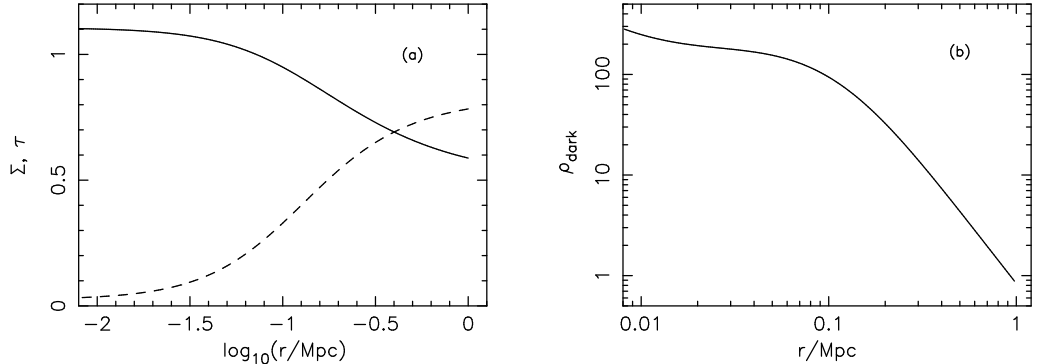


Figure 2. The  $k = 1$  cooling flow solution for A85; (a)  $\Sigma$  (dashed line) and  $\tau$  (solid line), (b) the density of the gravitating matter.

well-fit at radii greater than 20 kpc by a King model,

$$\rho_{\text{grav}} \propto \left[ 1 + \left( \frac{r}{0.11 \text{ Mpc}} \right)^2 \right]^{-1.25}. \quad (25)$$

Within this radius the density is poorly constrained. Although it appears to rise abruptly, only a small change in the slope of  $\xi$  would cause it to level off or even fall—all we know for sure is that the virial temperature becomes very small. Note also that there is only one bin within 20 kpc and this one is most likely to be affected by smoothing by the point-spread function, uncertain correction for excess absorption, etc.

The temperature of the gas is approximately constant outside the core radius, but drops by a factor of five in to 10 kpc. A temperature decline in the centre of clusters is typical of cooling flows observed by ASCA.

Note that the slope of the mass-deposition profile,  $\tau$ , is close to unity within the cooling radius,  $r_{\text{cool}} \approx 150$  kpc. This radius is not a special one for our solutions as we have assumed the cooling flow solution holds everywhere. For this reason the asymptotic slope of the gravitational density profile at large radii should be taken with a pinch of salt.

The corresponding solution for  $k = \infty$  is shown in Figure 3. There is no core in the gravitational mass profile in this case, with  $\rho_{\text{dark}}$  rising as  $r^{-2}$  all the way into 10 kpc. This is reflected in the temperature profile, however, which also rises by a factor of 5 between 150 and 10 kpc.

I conclude that A85 has a maximum core radius of 120 kpc and that self-similar solutions with low values of  $k$  provide more plausible temperature gradients than those with high values. If the mass profile has a constant-density core then  $k = 1$ , as predicted above.

## 5. Conclusions

- Cooling flows are known to be multiphase and the icm of clusters may well be multiphase throughout. This can affect mass measurements.

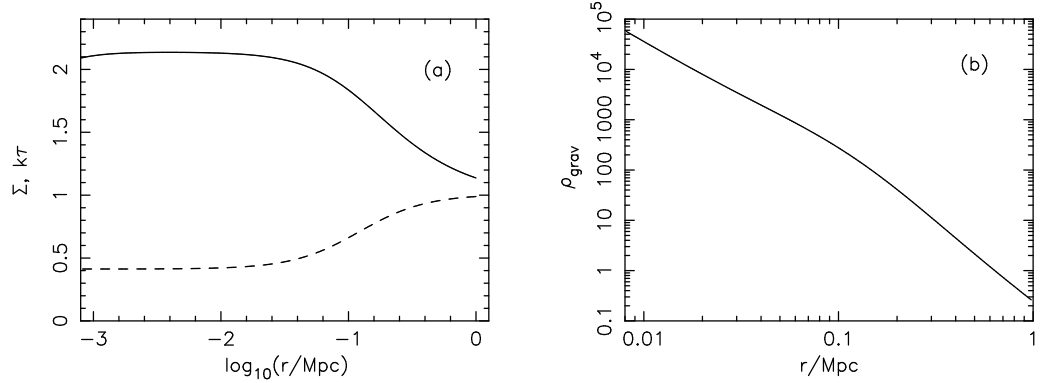


Figure 3. The  $k = \infty$  cooling flow solution for A85; (a)  $\Sigma$  (dashed line) and  $\tau$  (solid line), (b) the density of the gravitating matter.

- I have rederived the cooling flow equations for self-similar density distributions. Two of these in particular are expected to bound the behaviour of all possible flows.
- The steady-state cooling flow equations are *not* compatible with all conceivable emissivity profiles. Thus they can be used as a test of the theory.
- The solutions provide bounds on  $M(r)$  within the cooling radius and given  $k$  can be used to measure  $M(r)$ .

It would be very useful to apply the cooling flow models discussed here to galaxies and clusters with well-resolved cooling flows and especially those with measured mass-profiles or temperature gradients.

**Acknowledgments.** I would like to thank Clovis Peres for supplying me with the data for A85. This work was carried out while I was a Nuffield Foundation Science Research Fellow.

## References

- Gunn K. F., Thomas P. A., 1996, MNRAS, in press  
 Nulsen P. E. J., 1986, MNRAS, 221, 377  
 Thomas P. A., 1988a, in **NATO ASI** Cooling flows in clusters and galaxies, A. C. Fabian, Dordrecht: Kluwer, 361  
 Thomas P. A., 1988b, MNRAS, 235, 315  
 Thomas P. A., Fabian A. C., Nulsen P. E. J. N., 1987, MNRAS, 228, 973  
 Walker T. P., Steigman G., Schramm D. N., Olive K. A., Kang H. -S., 1991, ApJ, 376, 51  
 White D. A., Fabian A. C., 1995, MNRAS, 273, 72  
 White S. D. M., Navarro J. F., Evrard A. E., Frenk C. S., 1993, Nature, 366, 429